Complete Solutions Manual for Calculus of a Single Variable, Volume 1

Calculus

ELEVENTH EDITION

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CENGAGE Learning

ISBN-13: 978-1-337-27540-8
ISBN-10: 1-337-27540-9

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Boston, MA 02210
USA

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Printed in the United States of America
Print Number: 01 Print Year: 2017

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Preparation for Calculus

C H A P T E R P
Preparation for Calculus

Section P.1 Graphs and Models

1. To find the x-intercepts of the graph of an equation, let y be zero and solve the equation for x. To find the y-intercepts of the graph of an equation, let x be zero and solve the equation for y.

2. Substitute the x- and y-values of the ordered pair into both equations. If the ordered pair satisfies both equations, then the ordered pair is a point of intersection.

3. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

4. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

5. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

6. $y = x^3 - x$

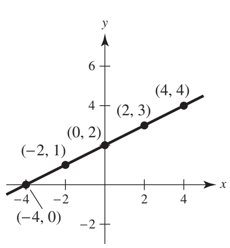
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

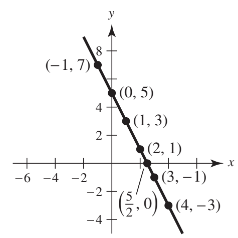
7. $y = \frac{1}{2}x + 2$

x	−4	−2	0	2	4
y	0	1	2	3	4



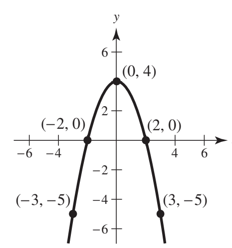
8. $y = 5 - 2x$

x	−1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	−1	−3



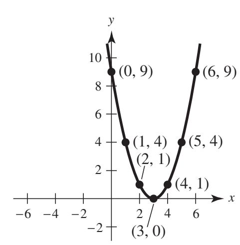
9. $y = 4 - x^2$

x	−3	−2	0	2	3
y	−5	0	4	0	−5



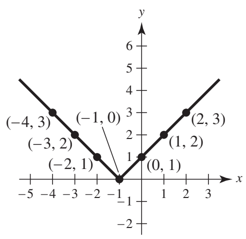
10. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



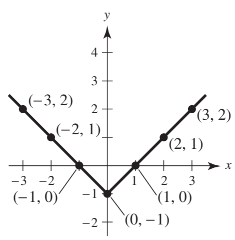
11. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



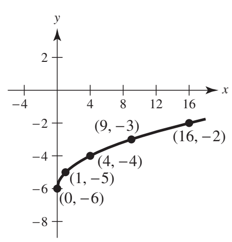
12. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



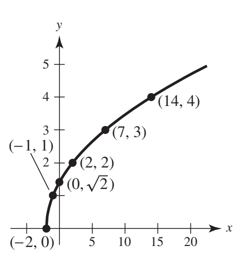
13. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



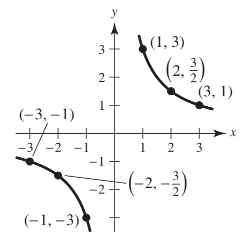
14. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



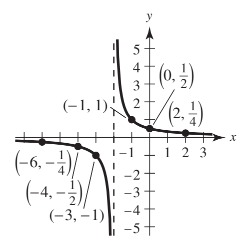
15. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

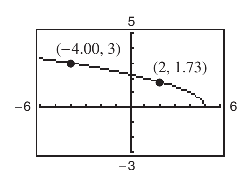


16. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



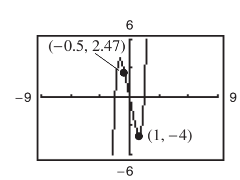
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5;\ (0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2};\ \left(\frac{5}{2}, 0\right)$

20. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3;\ (0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

22. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

24. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{\left[3(0) + 1\right]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

28. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

29. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6.$

30. $y = 9x - x^2$

No symmetry with respect to either axis or the origin.

31. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x.$

32. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

33. Symmetric with respect to the origin because $(-x)(-y) = xy = 4.$

34. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10.$

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

36. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

38. Symmetric with respect to the origin because

$$-y = \frac{(-x)^5}{4 - (-x)^2}$$
$$-y = \frac{-x^5}{4 - x^2}$$
$$y = \frac{x^5}{4 - x^2}.$$

39. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

40. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

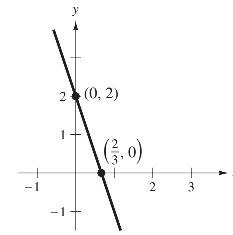
41. $y = 2 - 3x$

$y = 2 - 3(0) = 2,$ y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3},$ x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



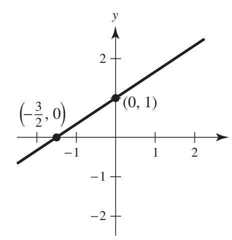
42. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1,$ y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2},$ x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



43. $y = 9 - x^2$

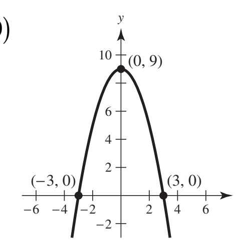
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



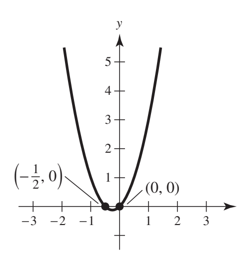
44. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



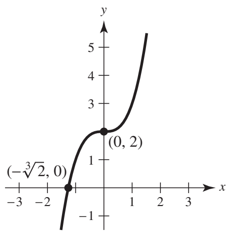
45. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



46. $y = x^3 - 4x$

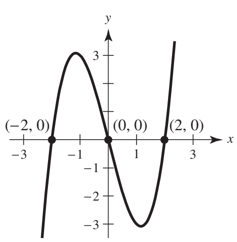
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \text{ } x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



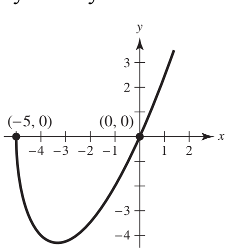
47. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



48. $y = \sqrt{25 - x^2}$

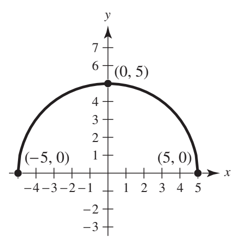
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \text{ } x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



49. $x = y^3$

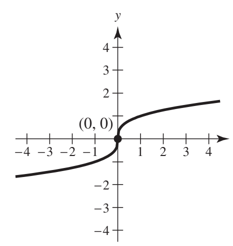
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



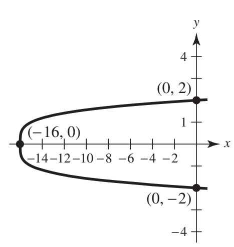
50. $x = y^4 - 16$

$$y^4 - 16 = 0$$
$$(y^2 - 4)(y^2 + 4) = 0$$
$$(y - 2)(y + 2)(y^2 + 4) = 0$$
$$y = \pm 2, \text{ } y\text{-intercepts}$$

$x = 0^4 - 16 = -16$, x-intercept

Intercepts: $(0, 2), (0, -2), (-16, 0)$

Symmetry: x-axis because $x = (-y)^4 - 16 = y^4 - 16$



51. $y = \dfrac{8}{x}$

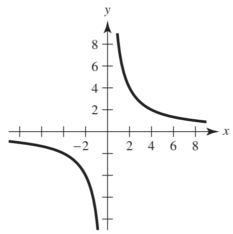
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



52. $y = \dfrac{10}{x^2 + 1}$

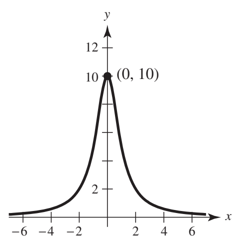
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



53. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

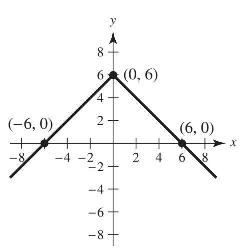
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



54. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

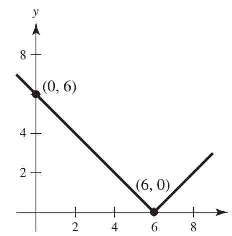
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



55. $3y^2 - x = 9$

$3y^2 = x + 9$

$y^2 = \frac{1}{3}x + 3$

$y = \pm\sqrt{\frac{1}{3}x + 3}$

$y = \pm\sqrt{0 + 3} = \pm\sqrt{3}$, y-intercepts

$\pm\sqrt{\frac{1}{3}x + 3} = 0$

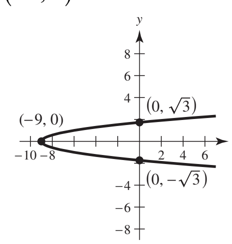
$\frac{1}{3}x + 3 = 0$

$x = -9$, x-intercept

Intercepts: $(0, \sqrt{3}), (0, -\sqrt{3}), (-9, 0)$

$3(-y)^2 - x = 3y^2 - x = 9$

Symmetry: x-axis



56. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

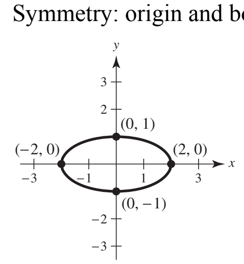
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$8 - x = 4x - 7$$
$$15 = 5x$$
$$3 = x$$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$$\frac{3x + 4}{2} = \frac{-4x - 10}{2}$$
$$3x + 4 = -4x - 10$$
$$7x = -14$$
$$x = -2$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$$-x^2 + 15 = 3x + 11$$
$$0 = x^2 + 3x - 4$$
$$0 = (x + 4)(x - 1)$$
$$x = -4, 1$$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1), (1, 14)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$$3 - x = (x - 1)^2$$
$$3 - x = x^2 - 2x + 1$$
$$0 = x^2 - x - 2 = (x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$$5 - x^2 = (x - 1)^2$$
$$5 - x^2 = x^2 - 2x + 1$$
$$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 16$

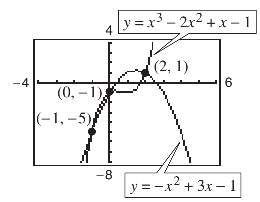
$x + 2y = 4 \Rightarrow x = 4 - 2y$

$$(4 - 2y)^2 + y^2 = 16$$
$$5y^2 - 16y + 16 = 16$$
$$y(5y - 16) = 0 \Rightarrow y = 0, \tfrac{16}{5}$$
$$x = 4 - 2(0) \Rightarrow x = 4$$
$$x = 4 - 2\left(\tfrac{16}{5}\right) \Rightarrow x = -\tfrac{12}{5}$$

Points of intersection: $(4, 0), \left(-\frac{12}{5}, \frac{16}{5}\right)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection:

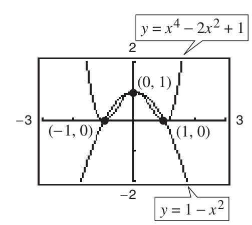
$(-1, -5), (0, -1), (2, 1)$

Analytically,

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2.$$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



Points of intersection:

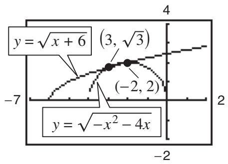
$(-1, 0), (0, 1), (1, 0)$

Analytically,

$$1 - x^2 = x^4 - 2x^2 + 1$$
$$0 = x^4 - x^2$$
$$0 = x^2(x + 1)(x - 1)$$
$$x = -1, 0, 1.$$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$

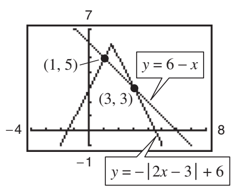


Points of intersection: $(-2, 2), (-3, \sqrt{3}) \approx (-3, 1.732)$

Analytically,
$$\begin{aligned} \sqrt{x + 6} &= \sqrt{-x^2 - 4x} \\ x + 6 &= -x^2 - 4x \\ x^2 + 5x + 6 &= 0 \\ (x + 3)(x + 2) &= 0 \\ x &= -3, -2. \end{aligned}$$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

$$|2x - 3| = x$$

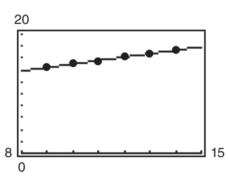
$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1$.

67. (a) Using a graphing utility, you obtain

$y = 0.58t + 9.2$.

(b)



The model is a good fit for the data.

(c) For 2024, $t = 24$:

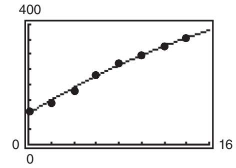
$y = 0.58(24) + 9.2 \approx 23.1$

The GDP in 2024 will be approximately \$23.1 trillion.

68. (a) Using a graphing utility, you obtain

$y = -0.35t^2 + 22.9t + 103$.

(b)



The model is a good fit for the data.

(c) For 2024, $t = 24$:

$y = -0.35(24)^2 + 22.9(24) + 103 \approx 451$

There will be approximately 451 million cell phone subscribers in 2024.

69.
$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

70. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$

$1 = 4k$

$k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$

$16 = 8k$

$k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$

$9 = 12k$

$k = \frac{9}{12} = \frac{3}{4}$

71. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at

$x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

72. Yes. If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

73. Yes. Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

74. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because $-y = (-x)^3 - (-x) = -x^3 + x$.

Symmetry with respect to the y-axis for $y = x^2 + 2$ because $y = (-x)^2 + 2 = x^2 + 2$.

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

75. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

76. True. $f(4) = f(-4)$.

77. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

78. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

1. In the form $y = mx + b$, m is the slope and b is the y-intercept.

2. No. Perpendicular lines have slopes that are negative reciprocals of each other. So, one line has a positive slope and the other line has a negative slope.

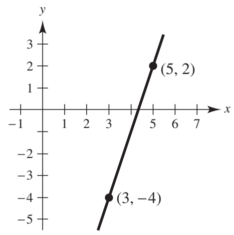
3. $m = 2$

4. $m = 0$

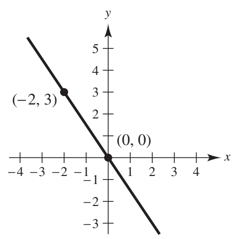
5. $m = -1$

6. $m = -12$

7. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

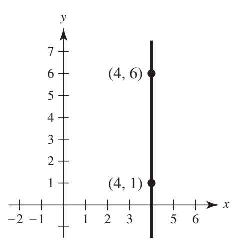


8. $m = \dfrac{3 - 0}{-2 - 0} = -\dfrac{3}{2}$



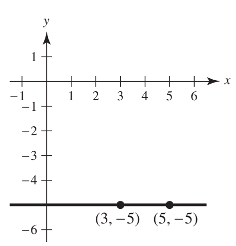
9. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

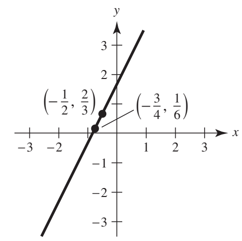


10. $m = \dfrac{-5 - (-5)}{5 - 3} = \dfrac{0}{2} = 0$

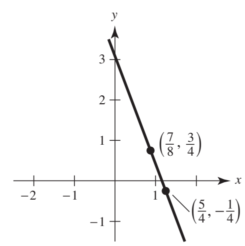
The line is horizontal.



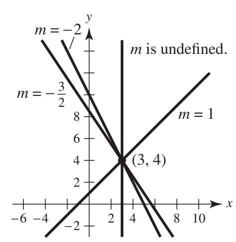
11. $m = \dfrac{\frac{2}{3} - \frac{1}{6}}{-\frac{1}{2} - \left(-\frac{3}{4}\right)} = \dfrac{\frac{1}{2}}{\frac{1}{4}} = 2$



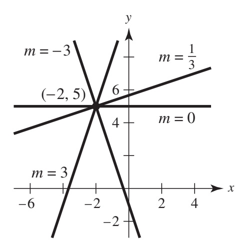
12. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$



13.



14.



15. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

16. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

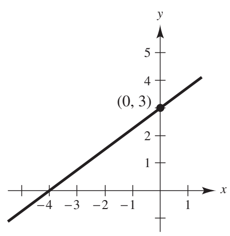
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

18. The equation of this line is

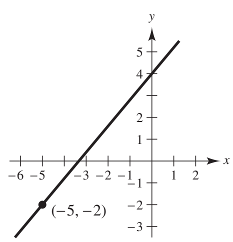
$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

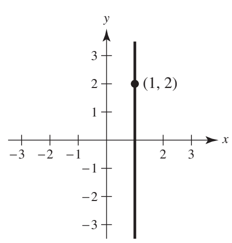
19.
$$y = \frac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



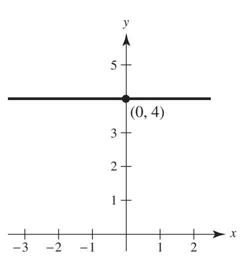
20.
$$y - (-2) = \frac{6}{5}[x - (-5)]$$
$$y + 2 = \frac{6}{5}(x + 5)$$
$$y + 2 = \frac{6}{5}x + 6$$
$$y = \frac{6}{5}x + 4$$
$$0 = 6x - 5y + 20$$



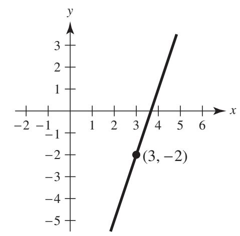
21. Because the slope is undefined, the line is vertical and its equation is $x = 1$.



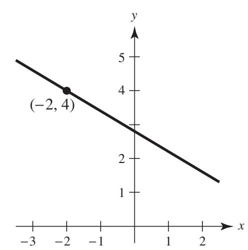
22.
$$y = 4$$
$$y - 4 = 0$$



23.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$



24.
$$y - 4 = -\frac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$

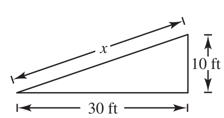


25.
$$\frac{6}{100} = \frac{x}{200}$$
$$100x = 1200$$
$$x = 12$$

Since the grade of the road is $\frac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

26. (a) $\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{1}{3}$

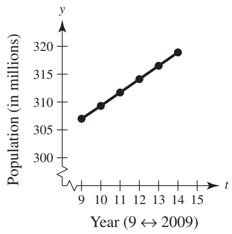
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

27. (a)



Slopes: $\dfrac{309.3 - 307.0}{10 - 9} = 2.3$

$\dfrac{311.7 - 309.3}{11 - 10} = 2.4$

$\dfrac{314.1 - 311.7}{12 - 11} = 2.4$

$\dfrac{316.5 - 314.1}{13 - 12} = 2.4$

$\dfrac{318.9 - 316.5}{14 - 13} = 2.4$

The population increased least rapidly from 2009 to 2010.

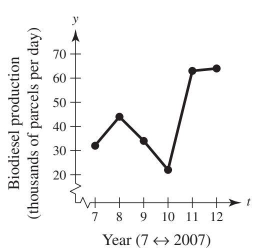
(b) $\dfrac{318.9 - 307.0}{14 - 9} = 2.38$ million people per year

(c) For 2025, $t = 25$:

$$\frac{P - 307.0}{25 - 9} = 2.38 \Rightarrow P = 2.38(16) + 307.0$$
$$\approx 345.1$$

The population of the United States in 2025 will be about 345.1 million people.

28. (a)



Slopes: $\dfrac{44 - 32}{8 - 7} = 12$

$\dfrac{34 - 44}{9 - 8} = -10$

$\dfrac{22 - 34}{10 - 9} = -12$

$\dfrac{63 - 22}{11 - 10} = 41$

$\dfrac{64 - 63}{12 - 11} = 1$

The population increased most rapidly from 2010 to 2011.

(b) $\dfrac{64 - 32}{12 - 7} = \dfrac{32}{5} = 6.4$ thousand barrels per day

(c) No. The production seems to randomly increase and decrease.

29. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

30. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is (0, 1).

31. $5x + y = 20$

$y = -5x + 20$

The slope is $m = -5$ and the y-intercept is (0, 20).

32. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

The slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

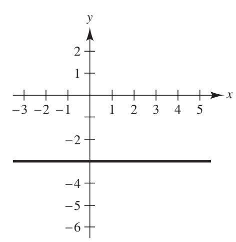
33. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

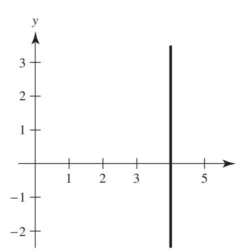
34. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

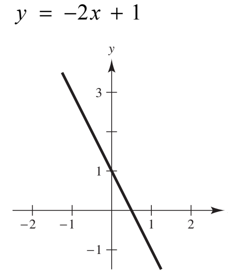
35. $y = -3$



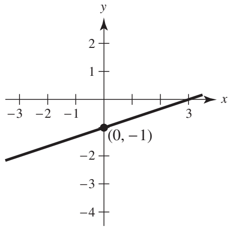
36. $x = 4$



37. $y = -2x + 1$

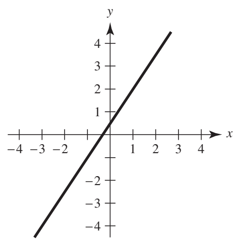


38. $y = \frac{1}{3}x - 1$



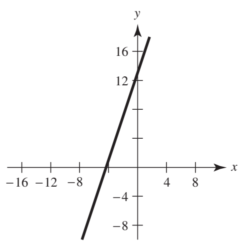
39. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



40. $y - 1 = 3(x + 4)$

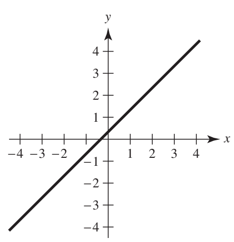
$y = 3x + 13$



41. $3x - 3y + 1 = 0$

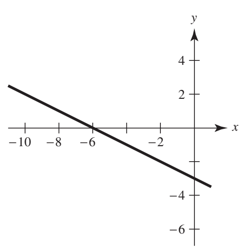
$3y = 3x + 1$

$y = x + \frac{1}{3}$



42. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

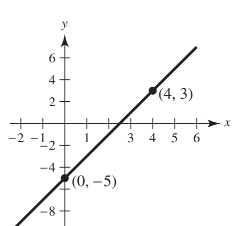


43. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$y - (-5) = 2(x - 0)$

$y + 5 = 2x$

$0 = 2x - y - 5$



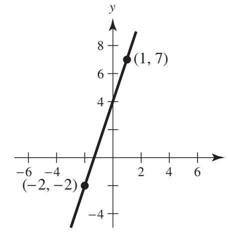
44. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y = 3x + 4$

$0 = 3x - y + 4$

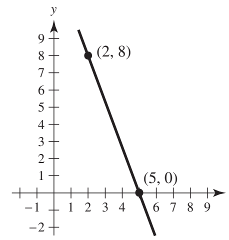


45. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$y - 0 = -\dfrac{8}{3}(x - 5)$

$y = -\dfrac{8}{3}x + \dfrac{40}{3}$

$8x + 3y - 40 = 0$

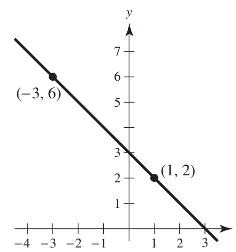


46. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

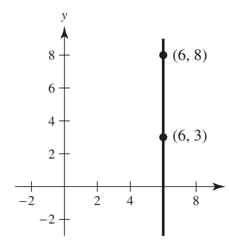
$x + y - 3 = 0$



47. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

The line is vertical.

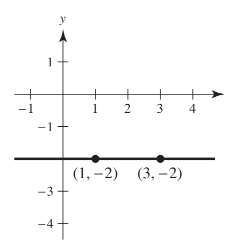
$x = 6$ or $x - 6 = 0$



48. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$y = -2$

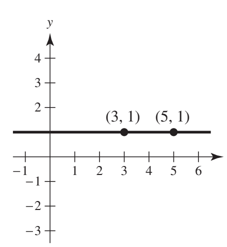
$y + 2 = 0$



49. $m = \dfrac{1 - 1}{5 - 3} = 0$

The line is horizontal.

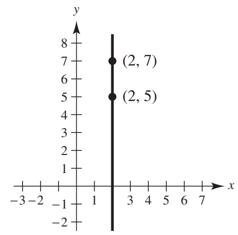
$y = 1$ or $y - 1 = 0$



50. $m = \dfrac{7-5}{2-2} = \dfrac{2}{0}$, undefined

The line is vertical.

$x = 2$ or $x - 2 = 0$



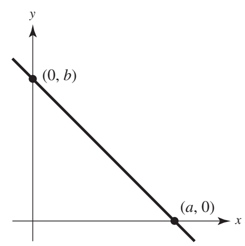
51. The slope is $\dfrac{1-b}{3-0} = \dfrac{1-b}{3}$.

The y-intercept is $(0, b)$. Hence,

$$y = mx + b = \left(\frac{1-b}{3}\right)x + b.$$

52. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$
$$\frac{b}{a}x + y = b$$
$$\frac{x}{a} + \frac{y}{b} = 1$$



53. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

54. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$
$$3x + y = -2$$
$$3x + y + 2 = 0$$

55. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$$\frac{9}{2a} + \frac{-2}{a} = 1$$
$$\frac{9-4}{2a} = 1$$
$$5 = 2a$$
$$a = \frac{5}{2}$$
$$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$
$$\frac{x}{5} + \frac{2y}{5} = 1$$
$$x + 2y = 5$$
$$x + 2y - 5 = 0$$

56. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$$\frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} = 1$$
$$-\frac{2}{3} + 2 = a$$
$$a = \frac{4}{3}$$
$$\frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} = 1$$
$$x - y = \frac{4}{3}$$
$$3x - 3y - 4 = 0$$

57. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

58. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

59. $x + y = 7$

$$y = -x + 7$$
$$m = -1$$

(a) $y - 2 = -1(x + 3)$

$$y - 2 = -x - 3$$
$$x + y + 1 = 0$$

(b) $y - 2 = 1(x + 3)$

$$y - 2 = x + 3$$
$$0 = x - y + 5$$

60. $x - y = -2$

$$y = x + 2$$
$$m = 1$$

(a) $y - 5 = 1(x - 2)$

$$y - 5 = x - 2$$
$$x - y + 3 = 0$$

(b) $y - 5 = -1(x - 2)$

$$y - 5 = -x + 2$$
$$x + y - 7 = 0$$

61. $5x - 3y = 0$

$$y = \tfrac{5}{3}x$$

$$m = \tfrac{5}{3}$$

(a) $y - \tfrac{7}{8} = \tfrac{5}{3}\left(x - \tfrac{3}{4}\right)$

$$24y - 21 = 40x - 30$$

$$0 = 40x - 24y - 9$$

(b) $y - \tfrac{7}{8} = -\tfrac{3}{5}\left(x - \tfrac{3}{4}\right)$

$$40y - 35 = -24x + 18$$

$$24x + 40y - 53 = 0$$

62. $7x + 4y = 8$

$$4y = -7x + 8$$

$$y = \frac{-7}{4}x + 2$$

$$m = -\frac{7}{4}$$

(a) $y + \dfrac{1}{2} = \dfrac{-7}{4}\left(x - \dfrac{5}{6}\right)$

$$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$$

$$24y + 12 = -42x + 35$$

$$42x + 24y - 23 = 0$$

(b) $y + \dfrac{1}{2} = \dfrac{4}{7}\left(x - \dfrac{5}{6}\right)$

$$42y + 21 = 24x - 20$$

$$24x - 42y - 41 = 0$$

63. The slope is 250.

$V = 1850$ when $t = 6$.

$$V = 250(t - 6) + 1850$$

$$= 250t + 250$$

64. The slope is -1600.

$V = 17{,}200$ when $t = 6$.

$$V = -1600(t - 6) + 17{,}200$$

$$= -1600t + 26{,}800$$

65. $m_1 = \dfrac{1 - 0}{-2 - (-1)} = -1$

$$m_2 = \frac{-2 - 0}{2 - (-1)} = -\frac{2}{3}$$

$m_1 \neq m_2$

The points are not collinear.

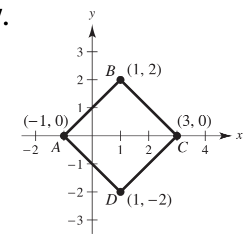
66. $m_1 = \dfrac{-6 - 4}{7 - 0} = -\dfrac{10}{7}$

$$m_2 = \frac{11 - 4}{-5 - 0} = -\frac{7}{5}$$

$m_1 \neq m_2$

The points are not collinear.

67.



The four sides are of equal length: $\sqrt{8} = 2\sqrt{2}$.

For example, the length of segment AB is

$$\sqrt{\left(1 - (-1)\right)^2 + (2 - 0)^2} = \sqrt{4 + 4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2} \text{ units.}$$

Furthermore, the adjacent sides are perpendicular because the slope of $\overline{AB}$ is $\dfrac{2 - 0}{1 - (-1)} = \dfrac{2}{2} = 1$, whereas the slope of $\overline{BC}$ is $\dfrac{2 - 0}{1 - 3} = -1$.

68. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$

$$y = \tfrac{1}{8}(5x + 4) = \tfrac{5}{8}x + \tfrac{1}{2}$$

(d) The slope must be $-\tfrac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$

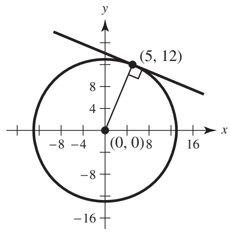
$$y = \tfrac{1}{2}(-5x + 4) = -\tfrac{5}{2}x + 2$$

(e) $a = \tfrac{5}{2}$ and $b = 3$.

$$\tfrac{5}{2}x + 3y = 4$$

$$5x + 6y = 8$$

69. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).

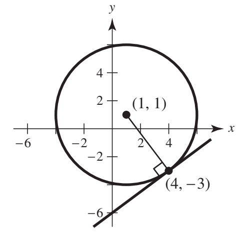


Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$
$$5x + 12y - 169 = 0.$$

70. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is $\frac{1 + 3}{1 - 4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$
$$0 = 3x - 4y - 24$$

71. (a) The slope of the segment joining (b, c) and $(a, 0)$ is $\frac{c}{(b - a)}$. The slope of the perpendicular bisector of this segment is $\frac{(a - b)}{c}$. The midpoint of this segment is $\left(\frac{a + b}{2}, \frac{c}{2}\right)$.

So, the equation of the perpendicular bisector to this segment is

$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right).$$

Similarly, the equation of the perpendicular bisector of the segment joining $(-a, 0)$ and $(a, 0)$ is

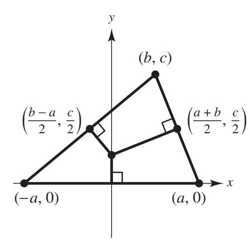
$$y - \frac{c}{2} = \frac{a - b}{-c}\left(x - \frac{b - a}{2}\right).$$

Equating the right-hand sides of each equation, you obtain $x = 0$.

Letting $x = 0$ in either equation yields the point of intersection:

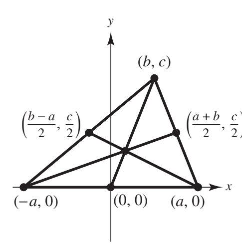
$$y = \frac{c}{2} + \frac{a - b}{c}\left(0 - \frac{a + b}{2}\right) = \frac{c^2}{2c} + \frac{b^2 - a^2}{2c} = \frac{c^2 + b^2 - a^2}{2c}.$$

The point of intersection is $\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right)$.



(b) The equations of the medians are:

$$y = \frac{c}{b}x$$
$$y = \frac{c/2}{\left(\frac{b - a}{2}\right) - a}(x - a) = \frac{c}{b - 3a}(x - a)$$
$$y = \frac{c/2}{\left(\frac{a + b}{2} + a\right)}(x + a) = \frac{c}{3a + b}(x + a).$$



Solving these equation simultaneously for (x, y), you obtain the point of intersection $\left(\frac{b}{3}, \frac{c}{3}\right)$.